

# **TIME-VARYING CROSS-SECTIONAL STOCK PORTFOLIO RETURNS ANALYSIS**

How do the underlying components driving stock portfolio returns develop over time and across market-beta sorted portfolios?

Bachelor's Thesis  
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## Abstract

I study the time-varying nature of stock portfolio returns in five market-beta sorted portfolios. By essentially regressing a nine-factor ordinary least squares (OLS) model on the portfolios, I uncovered differences between the underlying components of the five market-beta sorted portfolios. Subsequently, by performing a rolling-window regression on a constructed long-short portfolio, I exposed the time-varying nature of the underlying components driving portfolio returns. Uncovering the time-varying decomposition of the underlying components driving portfolio returns increases the potential predictability of the portfolio returns in the future. Eventually, increased predictability of portfolio returns could increase investors' ability to profit from market timing as well as allow them to hedge more efficiently.

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**Keywords** CAPM, Fama-French three-factor model, Fama-French five-factor model, momentum factor, credit spread, dividend yield, term spread, heteroscedasticity, autocorrelation, robust t-statistics.

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# 1. Introduction

By now there are countless studies in financial literature consisting of an abundance of evidence demonstrating that parameters such as means, variances and covariances of stock returns are time-varying and predictable (Yacine and Brandt, 2001). The importance of the Fama-French five factors has been cemented into literature throughout time. Additionally, there are some macroeconomic factors that have been proven to forecast stock portfolio returns to a high extent. Some of these factors include dividend yield, term spread, credit spread, momentum, bond yield and treasury bill yield (Yacine and Brandt, 2001).

Although a lot of research has been conducted regarding certain macroeconomic regressors and their effect on stock returns, a cross-sectional analysis across market-beta sorted portfolios is yet to be performed. A cross sectional analysis of how a combination of commonly known predictors of stock returns varies across portfolios sorted on market-beta can provide further insight into the nature of the relationship between the regressors and the returns of the stocks within these portfolios. Furthermore, performing a rolling-window regression on a constructed long-short portfolio exposes the time-varying nature of underlying components driving portfolio returns in high-beta portfolios versus low-beta portfolios.

Therefore, the focus of this research paper is to answer the research question:

**How do the underlying components driving stock portfolio returns develop over time and across market-beta sorted portfolios?**

As the model's objective is to explain portfolio returns, it is imperative to use some sort of asset pricing model in the regression. Therefore, the most relevant literature associated with this paper is the evolution of modern portfolio theory and the asset pricing model associated with it. Thus, the following asset pricing models are introduced, the capital asset pricing model (CAPM), the Fama-French three-factor model (FF3) and the Fama-French five-factor model (FF5), since they are the most relevant for the regression.

## **2. Literature Review**

### **2.1 Modern Portfolio Theory (MPT)**

Modern portfolio theory is a rather new area of study with the beginnings of it being credited to Harry Markowitz in a 1952 essay (Terin Miller). The portfolio optimization model Markowitz proposed in the essay outlines how investors can maximize the expected returns on their portfolio given the amount of risk they are willing to accept. In this model variance is used as a proxy for risk.

Therefore, modern portfolio theory is also termed mean-variance analysis. By implementing the model on a large amount of different accepted risk levels, the optimal portfolios for each level of variance are obtained, which form the efficient frontier (Terin Miller).

Subsequently, the efficient frontier paired with a risk-free investment yield a line called the capital market line. It is a line tangential to the efficient frontier and the intersection is named the tangent portfolio. Mean-variance dominance is achieved by choosing any portfolio on the newly formed capital market line. The capital market line implies all investable capital is invested in either the tangent portfolio ( $w$ ) or the risk-free asset ( $1-w$ ). If the investor wants to assume more risk than the tangent portfolio, it can be done by borrowing so that the weight in the risky tangent portfolio is above 1 ( $w > 1$ ). However, the model assumes investors can borrow at the risk-free rate, even though that seems quite unrealistic (Terin Miller).

The important assumption of MPT is of the asset pricing model used and it all begins with the capital asset pricing model (CAPM).

### **2.2 Capital Asset Pricing Model (CAPM)**

At the foundations of modern financial theory and especially modern portfolio theory lies the famous invention of the CAPM single factor model. The 1960s invention outlines a simple relationship for expected return and risk (Hunt).

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

The model imposes that the expected return on a specific asset ( $i$ ) is given by the risk-free rate plus the asset's beta times the market risk premium. Thus, the

model implies that the only risk related to an asset's expected return is captured by the CAPM beta (market-beta from now on). Mathematically, beta represents the ratio of the covariance of the stock's return and the market's return over the variance of the market. For so long (in the short history of modern finance) the CAPM model accurately explained the risk and return phenomena observed empirically (Hunt).

### 2.3 Fama-French Three-Factor Model (FF3)

However, when anomalies such as the small firm anomaly arose in empirical data, a new model was required to explain such inconsistencies. Therefore, in 1992, the Fama-French three-factor model was developed in response.

$$E(R_i) = R_f + \beta_{i,Mkt-RF}(E(R_m) - R_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML)$$

The three-factor model added both a size factor (SMB) as well as a value factor (HML), which quite accurately explain the anomalies present in the data up until then (Hunt).

### 2.4 Fama-French Five-Factor Model (FF5)

More recently in 2015, the Fama-French five factor model has emerged (Hunt).

$$E(R_i) = R_f + \beta_{i,Mkt-RF}(E(R_m) - R_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML) + \beta_{i,RMW}E(RMW) + \beta_{i,CMA}E(CMA)$$

The five-factor model has two additional factors attempting to capture the profitability and investment effects present in data (Hunt).

### 2.5 Additional Variables

Additionally, to the famous Fama-French factors above, for example, Avramov (1999) uses term spread, earnings yield, treasury bill yield, book-to-market ratio and dividend yield to predict expected returns in a single period context.

Campbell and Viceira (1999), Balduzzi and Lynch (1999) and Barberis (2000) use the dividend yield to predict expected returns in a multiperiod context. Brandt (1999) uses term spread, default spread, lagged return and dividend yield in a multiperiod context. Brennan, Schwartz and Lagnado (1997) use the bond yield, dividend yield and treasury bill yield. Finally, Lynch (2000) uses the term spread and dividend yield (Yacine and Brandt, 2001).

Together with the famous Fama-French five factors, I will additionally include macroeconomic variables such as a momentum factor, credit spread, dividend yield and term spread in my regression. These additional factors add insight into how the macroeconomic environment affects the returns on the stocks that comprise the portfolios being analyzed. They are chosen because they seem to be a quite common denominator across the existing literature.

### 3. Data and Methodology

#### 3.1 Origin of data

Firstly, the market-beta sorted portfolio returns used as the dependent variables in my regression were obtained from the Fama-French data library, which has constructed its portfolios from the CRSP database (Description of Fama/French Factors). Additionally, the regressors such as the Fama-French 5-factors as well as the momentum factor have been obtained from the same source (Description of Fama/French Factors). Subsequently, the data to construct the credit spread and the term spread regressors originates from the St. Louis FRED (Moody's Seasoned Aaa Corporate Bond Yield, Moody's Seasoned Baa Corporate Bond Yield, 1-Year Treasury Constant Maturity Rate and 10-Year Treasury Constant Maturity Rate). Finally, the data used as a proxy for the dividend yield regressor is from Multpl (S&P 500 Dividend Yield by Month).

#### 3.2 Regression

In order to perform a multi-variable linear regression on the five sorted portfolio returns, I regressed the following ordinary least squares (OLS) linear model:

$$R_{i,t} - R_{f,t} = \alpha_t + \beta_{Mkt-RF}(R_{m,t} - R_{f,t}) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{RMW}RMW_t + \beta_{CMA}CMA_t \\ + \beta_{MOM}MOM_t + \beta_{Cred}Cred_t + \beta_{Div}Div_t + \beta_{Term}Term_t + \varepsilon_t$$

In the model above, the dependent variable on the left-hand-side represents the excess return on the specific market-beta sorted portfolio. The five market-beta sorted portfolios that I have used are “formed on univariate market-beta at the end of each June using NYSE breakpoints. Beta for June of each year is estimated using the preceding five years (minimum of two) of past monthly returns” (Detail for Portfolios Formed on Market-Beta).

In the model above, the independent variables on the right-hand-side of the equation represent the regressors used in the model: market excess return (Mkt-RF), small-minus-big (SMB, size factor), high-minus-low (HML, value factor), robust-minus-weak factor (RMW, profitability factor), conservative-minus-aggressive factor (CMA, investment factor), momentum factor (MOM), credit spread (Credit), dividend yield (Div) and term spread (Term) (Description of Fama/French Factors).



### 3.3 Calculating Each Regressor and Its Importance

Mkt-RF:

The market excess return is the “value weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month  $t$ , good shares and price data at the beginning of  $t$ , and good return data for  $t$  minus the one-month Treasury bill rate (from Ibbotson Associates) (Description of Fama/French Factors).” The importance of the market premium factor stems from the foundation of asset pricing models, i.e. from the CAPM and is also the most important factor in other asset pricing models (Market Risk Premium).

SMB:

“Small-minus-big is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios (Description of Fama/French Factors).” The Fama-French SMB factor is an important variable to include as a regressor as it captures the size effect in the market through analysis of characteristics such as B/M ratio, operating profitability as well as size/investment portfolios (Fama and French 2015).

HML:

“High-minus-low is the average return on the two value portfolios minus the average return on the two growth portfolios (Description of Fama/French Factors).” The Fama-French HML factor is also an important variable to include as a regressor because it captures the value effect in the market through analysis of a value portfolio’s returns compared to a growth portfolio’s returns (Fama and French 2015).

RMW:

“Robust-minus-weak is the average return on the robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios (Description of Fama/French Factors).” The Fama-French RMW factor is also a useful variable to include as a regressor since it captures the

profitability effect in the market through analysis of firms' operating profitability (Fama and French 2015).

CMA:

"Conservative-minus-aggressive is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios (Description of Fama/French Factors)." The Fama-French CMA factor is also a vital variable to include as a regressor as it captures the investment effect in the market through analysis of firms' investment strategies (Fama and French 2015).

MOM:

"Momentum is the average return of two high prior return portfolios minus the average return on the two low prior return portfolios (Description of Fama/French Factors)." The Fama-French MOM factor is a crucial variable to include as a regressor as it mirrors the recent developments within the market. Its importance is well known as it is a part of the established Carhart four-factor model (Carhart 4 Factor Model).

Credit Spread:

The credit spread is calculated as Moody's BAA rated corporate bond yield minus Moody's similar maturity AAA rated corporate bond yield (Moody's Seasoned Aaa Corporate Bond Yield and Moody's Seasoned Baa Corporate Bond Yield). Credit spread is an important additional regressor on top of the Fama-French five factors as it exposes the attitudes of investors towards firms with different credit ratings. Investors' attitudes towards firms are often shaped by the expected probability of the firm to default on its bonds. The attitudes reflected by the yield of the firms' bonds affect total market liquidity and thus influence stock returns as well (What is Credit Spread?).

Dividend Yield:

"The dividend yield is the sum of dividends paid on the S&P index over the past 12 months divided by the current level of the index" (Yacine and Brandt 2001). The S&P 500 dividend yield is used as a proxy for the dividend yield regressor

(S&P 500 Dividend Yield by Month). Dividend yield is a well-established predictor of stock returns as it mechanically makes up a part of stock returns (the sum of dividend yield and capital gain). Therefore, the correlation between dividend yield and stock return is very high and near 1, which makes it a useful regressor to include in the model (Timmerman).

#### Term Spread:

The term spread is calculated as the constant maturity 10-year US treasury yield minus the constant maturity 1-year US treasury yield (1-Year Treasury Constant Maturity Rate and 10-Year Treasury Constant Maturity Rate).

Utilizing term spread as a regressor in the model highlights investors' attitudes towards the future, which is given by the term structure of US treasury yields. Investors attitudes and expectations about future interest rates significantly influence the entire economy as well as stock market returns (A Note on the Term Spread).

#### Risk-free rate:

The return for a 1-month US treasury has been used as a proxy for the risk-free rate (Description of Fama/French Factors). The return of the 1-month US treasury is a valid proxy for the risk-free rate because it is the closest approximation to the theoretical return on an investment with zero risk, since it is fully backed by the US government (Risk Free Rate). Furthermore, the duration of one month matches the investment horizon of the monthly returns on the market-beta sorted portfolios making it the most suitable treasury to use as a proxy (Risk Free Rate).

### **3.4 Robustness Checks**

The regression results were adjusted for heteroscedasticity and autocorrelation. Heteroscedasticity occurs when subpopulations of the original data have different variabilities to each other (Frost). This means that the variance of the data is not constant across the whole sample of data used. This is a major problem since fluctuating variances can invalidate certain conclusions drawn from data by making them insignificant after adjusted for heteroscedasticity.

Autocorrelation is the correlation of observations as a function of the time lag between them (Autocorrelation – Statistics Solutions). This means that the previous observations predict future observations to a high extent. This is also a major problem since self-predicting observations can also invalidate certain conclusions drawn from data by making them insignificant after adjusted for autocorrelation.

The maximum time-period regression as well as the subsequent rolling window regressions were carried out in a way to obtain Huber-White HC4 standard errors (Robust Standard Errors). Both a Breusch-Godfrey test as well as a Breusch-Pagan test were performed on the data set. The results of the tests showed a statistically significant presence of both autocorrelation and heteroscedasticity respectively. Therefore, in order to calculate robust t-statistics for the coefficients, firstly, a heteroscedasticity and autocorrelation consistent covariance matrix is obtained. The diagonal of such a matrix consists of the variances of each regressor. Taking the square root of the diagonal of the matrix yields robust standard errors. Then dividing the coefficients found in the regression by the robust standard errors results in the robust t-statistics. Thus, the resulting t-statistics are robust to heteroscedasticity and autocorrelation.

### 3.5 Performing the Rolling Window Regression

In essence, performing the multi-variable regression outlined earlier with a rolling window of 5 years for all of the five market-beta sorted portfolios yields a three-dimensional output table. The important statistics resulting from the above regression are indeed the coefficient of each regressor as well as the robust t-statistic associated with each coefficient, indicating the significance of the obtained coefficient.

Originally, the data contained 673 data points of monthly observations from (1/7/1963 – 1/7/2019), however applying a 5-year rolling window reduces the amount of output data points to 613 (1/7/1968 – 1/7/2019). For example, the first rolling window is from months 1-60, then 2-61 and 3-62 until 613-672. Now, with nine regressors and a constant (error term,  $\epsilon$ ), a 613 data point time series and five portfolios, the resulting output table is essentially a  $10 \times 613 \times 5$  three-dimensional table.

Technically, there are two of these  $10 \times 613 \times 5$  tables, since one is for the coefficients and one is for the robust t-statistics associated with the coefficients. Finding an effective way to display all of the results in limited space is challenging. Therefore, I have constructed a long-short excess return portfolio, which is equal to the returns of the high 20 market beta sorted portfolio minus the returns of the low 20 market beta portfolio minus the risk free-rate. By demonstrating the time-varying nature of the underlying components driving portfolio returns with a long-short portfolio, I am able to condense the data and thus analyze it more effectively.

## 4. Results

**4.1 Table I: Maximum Time Period Regression Output**

	Market Beta Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
Panel Regression: $R_{i,t} - R_{f,t} = \alpha_i + \beta_{Mkt-RF}(R_{m,t} - R_{f,t}) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{RMW}RMW_t + \beta_{CMA}CMA_t + \beta_{MOM}MOM_t + \beta_{Cred}Cred_t + \beta_{Div}Div_t + \beta_{Term}Term_t + \epsilon_t$										
	Coefficients/Betas ( $\beta$ )					Robust t( $\beta$ )				
Constant	-0.276	0.081	0.047	0.246	0.079	-1.125	0.406	0.260	1.080	0.267
Mkt-RF	0.816	0.984	1.073	1.162	1.301	<b>40.671</b>	<b>75.422</b>	<b>74.167</b>	<b>51.671</b>	<b>40.780</b>
SMB	-0.195	-0.071	0.061	0.246	0.523	<b>-6.403</b>	<b>-2.737</b>	<b>2.692</b>	<b>7.161</b>	<b>11.912</b>
HML	0.077	0.058	0.079	0.133	-0.010	1.955	1.883	<b>2.389</b>	<b>2.785</b>	-0.126
RMW	0.261	0.288	0.252	0.085	-0.201	<b>4.052</b>	<b>4.751</b>	<b>7.153</b>	1.278	<b>-3.645</b>
CMA	0.333	0.149	-0.011	-0.171	-0.289	<b>5.641</b>	<b>3.310</b>	-0.243	<b>-3.252</b>	<b>-3.406</b>
MOM	0.038	0.020	-0.047	-0.060	-0.126	1.320	0.880	<b>-1.968</b>	<b>-2.461</b>	<b>-4.831</b>
Credit	0.077	0.218	0.079	-0.102	-0.170	0.519	1.665	0.604	-0.576	-0.532
Div	0.046	-0.104	-0.040	-0.055	0.068	0.598	-1.659	-0.737	-0.734	0.662
Term	-0.007	-0.044	-0.053	-0.037	-0.084	-0.109	-0.908	-1.046	-0.640	-1.081
	Low	2	3	4	High					
R-Squared	0.8395	0.9407	0.9466	0.9412	0.9187					
Adjusted R-Squared	0.8373	0.9399	0.9459	0.9404	0.9176					
F-Statistic	385.2	1168.1	1305.3	1179.6	832.8					

\*significant t-statistics shown in **bold**

Table I exhibits the cross-sectional regressions of the whole time period (1/7/1963 – 1/7/2019) of the five market-beta sorted portfolios and the panel regression including the nine regressors and a constant term.

The f-statistic tests whether all coefficients are equal to zero in the model. The extremely high significance shows that all of the portfolios are highly affected/explainable by some combination of the regressors used in the model.

### 4.1.1 Significant Trends

Clearly, the most significant regressor in each portfolio by magnitude of coefficient and robust t-statistic is the market premium factor (Mkt-RF). The coefficient for the market premium factor increases consistently from the low beta portfolio to the high beta portfolio. This is a logical outcome as the market-betas originally used to sort the portfolios are essentially a measure of the same exposure (the exposure of the portfolio to the theoretical market portfolio).

Furthermore, the coefficients of SMB also increase relatively consistently with increasing market-beta. Simultaneously, their robust t-statistics all indicate that the results are of high significance. This trend is also an intuitive result as often

small cap firms have a larger market-beta due to the higher volatility in their returns (Fact, Fiction and the Size Effect).

Subsequently, the CMA factor is negatively correlated with market beta as the coefficients decrease with increasing market beta. Again, this occurs with all of the robust t-statistics of the coefficients being significant, except for quintile 3. This trend indicates that firms with higher market-beta (usually small cap firms) invest more aggressively. Once again, this finding is a logical outcome as small cap growth firms often have to invest more in order to eventually achieve significant sales growth and the maturity stage of the company's life cycle (What is the Difference between Large Cap & Small Cap Stocks?).

A slightly weaker trend occurs with the RMW regressor, since the coefficients decline almost consistently from the low-beta portfolio to the high-beta portfolio. Apart from the increase from the first portfolio to the second, the trend in RMW is declining with increasing market-beta. Together with significant robust t-statistics for all but one of the portfolios, this makes for a relatively significant trend. The reasoning behind this trend is also quite sound since the trend is implying that firms with higher market-betas (usually small cap firms) have weaker operating profitability. It is expected that a smaller growth company in the early stage of its company life cycle is more likely to be making losses than a mature company (Business Life Cycle).

Another relatively apparent trend is the consistent decline in the coefficient of the MOM regressor. This paired with three out of the five portfolios having a significant robust t-statistic make the trend somewhat noteworthy. The trend implies that higher beta portfolios have a negative loading to the momentum factor, which means that they have been among the worst performing stocks in the past month. Again, this implication can be explained by the fact that usually high-beta firms (small cap firms) make losses in the early stage of their company life cycle and thus are reflected by a negative loading to the momentum factor (Business Life Cycle and Fitzgerald).

#### 4.1.2 Lack of Significance in Trends

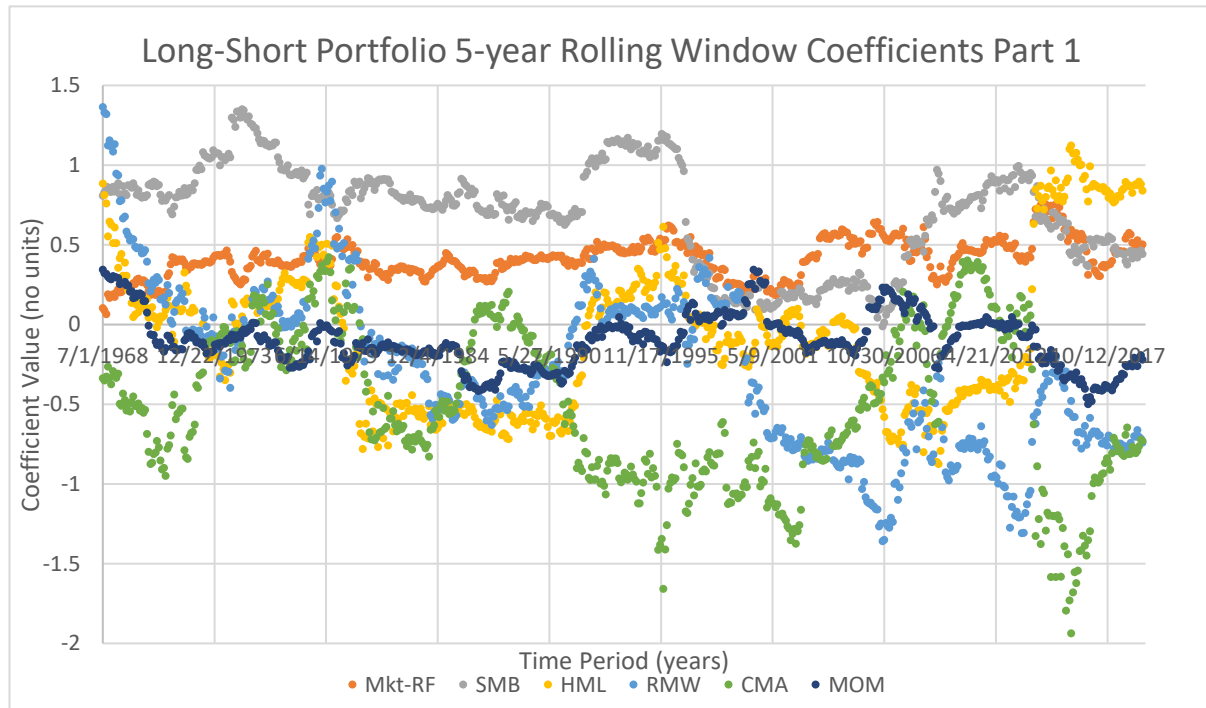
The rest of the regressors, HML, Credit, Div and Term, do not exhibit any apparent trends of consistently increasing or decreasing from the low-beta portfolio to the high-beta portfolio. The lack of significance is also highlighted by the insignificance of the robust t-statistics associated with almost all of the regressors for almost all of the portfolios. The only regressor out of these that exhibits slight significance is the HML regressor as it has a robust t-statistic for two of the five portfolios. The explanation of this is that the HML regressor would be significant in a model with less regressors, for example the FF-3 model. However, since more regressors have been included in the model, the same characteristics of firms that constitute HML have been caught by some other regressors such as RMW and CMA, making the HML regressor redundant in the model (Fama and French).

The most important regressor that lacks significance in the model is the inconsistent development of the constant term. This means that the constant term does not increase or decrease consistently from the low-beta portfolio to the high-beta portfolio. More importantly, the magnitudes of the robust t-statistics indicate that none of the constant terms are significant. Essentially, this means that the regression used in Table 1 accounts for almost all of the portfolios' returns it is regressed against, leaving no significant residual return unexplained.

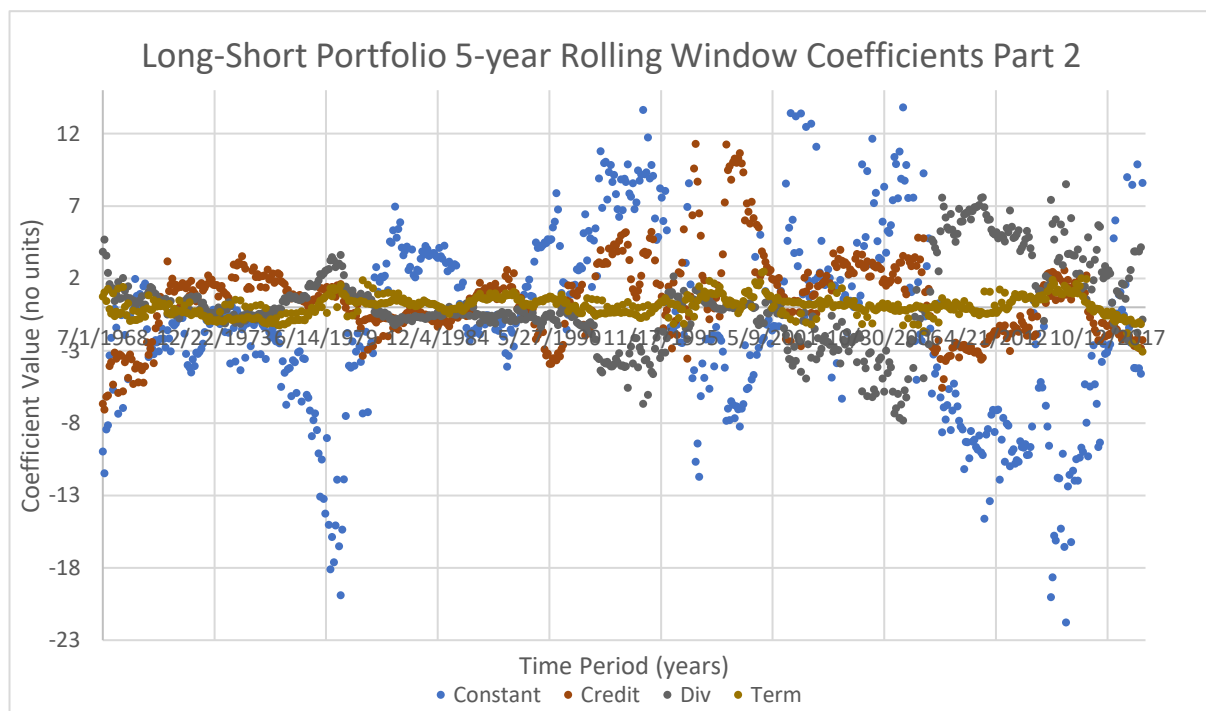
This is furthermore enhanced by the r-squared values of each regression being extremely high (between 90 and 95%, except for the low-beta portfolio, which was slightly lower). Even when considering the adjusted r-squared values instead of the original r-squared values, the regressors explain the data to a very high degree, since the adjusted r-squared values are only marginally lower than the original r-squared values. However, a limitation of this is that adjusted r-squared values might not penalize the addition of new regressors to the model harshly enough and because of this the adjusted r-squared values are only marginally lower than the original r-squared values. Therefore, in future tests a harsher penalty can be attributed to increasing the number of regressors.



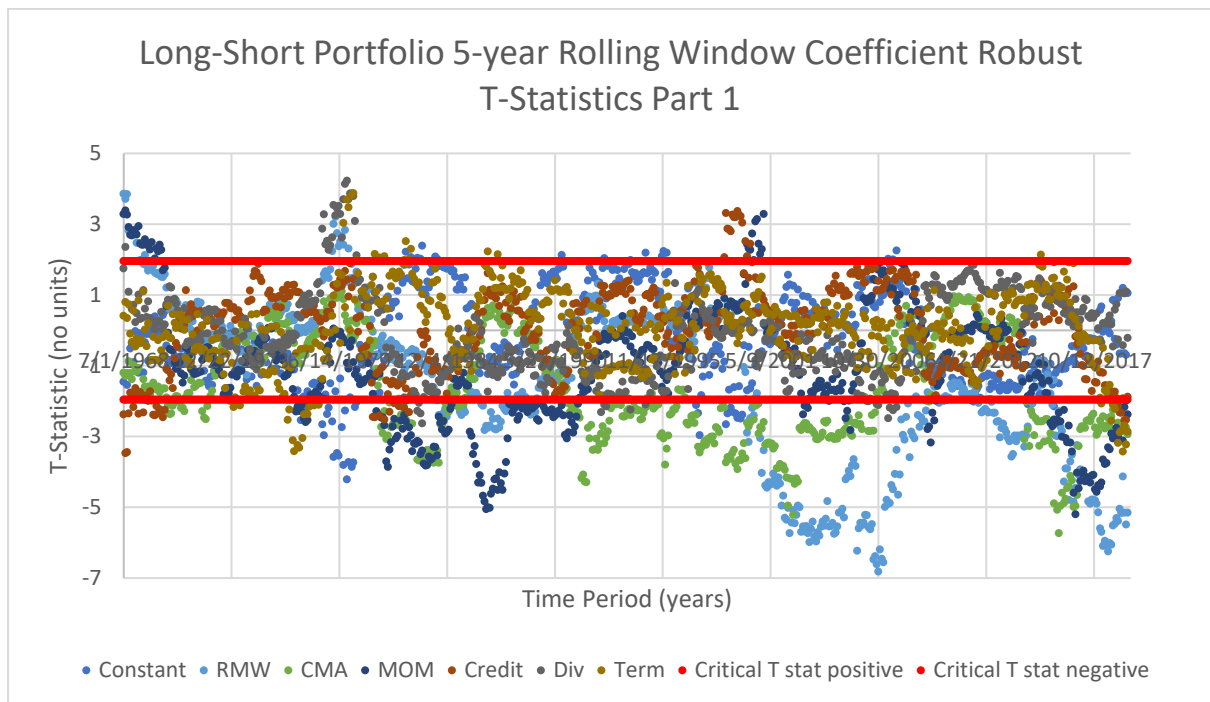
**4.2 Figure 1: Long-Short Portfolio 5-Year Rolling Window Coefficients Part 1**



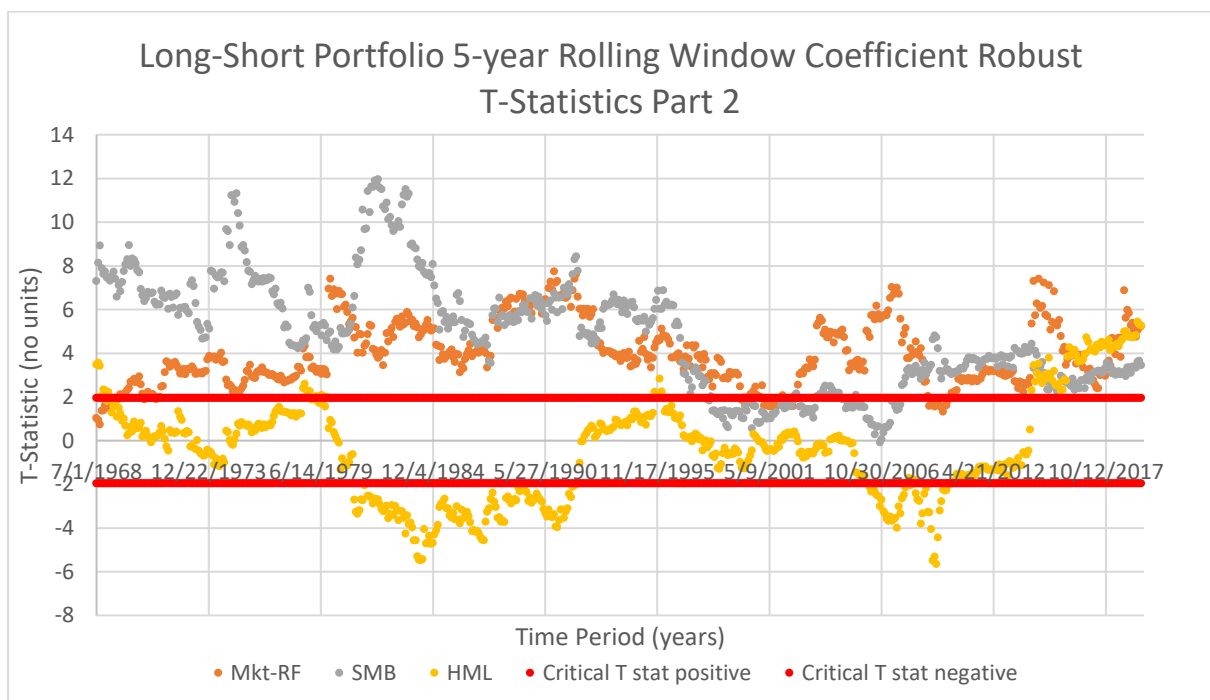
**4.3 Figure 2: Long-Short Portfolio 5-Year Rolling Window Coefficients Part 2**



**4.4 Figure 3: Long-Short Portfolio 5-Year Rolling Window Robust T-Statistics Part 1**



**4.5 Figure 4: Long-Short Portfolio 5-Year Rolling Window Robust T-Statistics Part 2**



## **5. Analysis and Application of Results**

### **5.1 Analysis**

Firstly, the data in Figure 1 and Figure 2 has been separated into two different graphs since if they were in the same graph the data from Figure 2 would enlarge the axis in a way to make the variations in Figure 1 seem meaningless. The same reason applies for why the data in Figure 3 and Figure 4 have been separated.

#### **5.1.1 Time-Varying Fluctuations**

The most important observations regarding Figure 1 and Figure 2 are that there are periods with more fluctuations and periods with less fluctuation.

Furthermore, during these periods of higher variability most of the coefficients tend to vary a lot. Therefore, there must be some common underlying factor or event affecting the variability of all of the factors simultaneously. For example, in both figures, the periods 1975-1982, 1991-2000 and 2006-2018 exhibit high variability compared to the other periods of time included in the figure.

Analyzing the high fluctuation of coefficients in the time period 1975-1982 from a significant event's standpoint indicates that it could most likely be associated with the 1970s energy crisis as well as the Latin American debt crisis (A History of the Past 40 Years in Financial Crises). This is a relatively rational explanation as energy prices are extremely volatile and especially during this time period the US economy was still heavily reliant on the manufacturing industry where energy consumption is high. Therefore, with high exposure to volatile energy prices, the underlying components explaining portfolio returns will vary significantly.

Similarly, the high fluctuation of coefficients in the time period 1991-2000 could most likely be related to the buildup and burst of the dot-com bubble crisis (A History of the Past 40 Years in Financial Crises). This is a very reasonable explanation since the dot-com bubble originated from the US and afterward affected financial markets all over the world as well. Again, this crisis affected

especially technology stocks in the US, but it also affected the whole market in general since most equities seemed to be overpriced.

Finally, the high fluctuations of coefficients in the time period 2006-2018 can most probably be attributed to the start of the financial crisis, followed by the deepening of the financial crisis, which eventually led to the European sovereign debt crisis (A History of the Past 40 Years in Financial Crises). Furthermore, after this the macroeconomic environment has been unlike any other seen previously with a negative interest rate environment. Again, these highly unusual and turbulent times led to the underlying components driving portfolio returns to vary significantly.

### **5.1.2 Interesting Observation**

An interesting observation is also the evolution of the HML and the RMW regressors in Figure 1. They vary in almost perfect lock-step from 1968-1999, however after that they begin to differ a lot. They also alternate between being positive and negative across the time period, which seems to suggest that the composition of these market-beta portfolios has changed tremendously over time. For example, when the HML coefficient changes from positive to negative it seems to imply that originally value firms had a higher market-beta than growth firms, but then growth firms obtained a higher market-beta than value firms.

### **5.1.3 Consistency & Inconsistency**

The most consistent coefficient across the entire time period is the Mkt-RF factor. As shown in Figure 1, it is the most consistent because it is constantly between a range of 0 and 0.9 the entire time and near 0.5 for most of the time period. This is an important observation because in order to have a reliable model, it is crucial to try to have the most important variable in the model to be as constant as possible throughout the time period.

The most varying coefficients are shown in Figure 2, where especially the constant, Credit and Div regressors vary to a high extent. Most notably, Figure 2 displays the ceaseless variability of the constant term across the whole time period. The ceaseless variability indicates that the model used in the rolling window regression does not constantly over- or underestimate the returns on the

long-short portfolio. This is an encouraging result since unexplainable persisting positive or negative returns indicate that there is some trend in the data that the model does not account for. Essentially, this means that a regressor or multiple regressors are missing from the model. However, the regressors used in this rolling window regression produce results explain more than 90% of the variability of the portfolio returns (which was shown by the adjusted r-squared of the maximum time period regression earlier). Therefore, the statistical error of underfitting does not occur in my rolling window regression.

#### **5.1.4 Significance & Insignificance**

Bearing in mind Figure 3 and Figure 4, most of the fluctuations of the coefficients fall within the insignificant robust t-statistic range for almost the whole time period. For example, basically all of the coefficients of the Credit, Div and Term regressors were in the insignificant region of the robust t-statistic graph for the whole time period. The remaining regressors in Figure 1, the constant, RMW, CMA and MOM, only exhibited statistical significance in a minority of cases within the whole time period.

However, this is mainly because performing a rolling window of 5 years on monthly data only supplies the regression with 60 data points on which to perform the regression. With a small number of data points, it is harder to get robust t-statistics since the probability that the data occurred by chance is a lot higher. On the contrary, with a larger amount of data points, e.g. a 10-year rolling regression, some of the variability is absorbed by the increasing amount of data points. Therefore, with more data points there appears to be less time-varying nature to the underlying components driving portfolio returns. Thus, eventually it is a trade-off between the degree of statistical significance of the coefficients and the degree of variability (time-varying nature) of the coefficients.

Figure 4 on the other hand shows how the Mkt-RF, SMB and HML regressors were statistically significant for the majority of the time period. This means that the coefficients obtained for these regressors in the rolling window regression do have significant meaning, even though they were obtained from a sample with so few data points.

## **5.2 Application of Results**

The results I obtained, especially from Table I are in line with previous results from Fama and French 2015. They are in line because most of the portfolios in the maximum time period regression (Table I) exhibited significant robust t-statistics to all of the five Fama-French factors.

Table I contradicts previous results obtained by Campbell and Viceira (1999), Balduzzi and Lynch (1999) and Barberis (2000) since dividend yield was not even close to being significant in any of the portfolios over the whole time period. Furthermore, the Div regressor was very rarely statistically significant in the rolling window regression shown in Figure 3. This could be due to the fact that the exposure is caught by other regressors, which would imply a statistical error of over-fitting the model.

The results in Table I also contradict previous results obtained by Brandt (1999) since credit spread and term spread were not significant in any of the portfolios over the whole time period. However, in the rolling window regression they were significant at times, so this could be a statistical error of over-fitting the model or just due to the whole time period neutralizing the more extreme data points occurring less frequently.

Although, my results were in line with some previous studies, it also contradicted others. Therefore, it is not that reliable of a model to be using to predict expected returns on stock portfolios. Thus, in order to profit off of market timing and hedge more efficiently, a more consistent and accurate model would need to be discovered.

## **5.3 Limitations**

Although the maximum time period regression showed in Table I exposes the underlying components driving the portfolio returns of the five market-beta sorted portfolios and the rolling window regression uncovers the time-varying nature of these underlying components to an extent, the actual reason for the why the components vary is not certain from this study. Therefore, only the fact that the components are time-varying is uncovered. However, the cause of the

time-varying nature is just attributed to significant financial crises that could have affected the components during the periods of high fluctuation.

Another limitation of this study includes the sole analysis of long-short portfolio during the rolling-window regression. Although, the long-short portfolio highlights the difference between the low market-beta and high market-beta portfolios, the incremental change from one portfolio to another during the rolling window could have been of interest. However, as mentioned earlier due to inconvenience and lack of space, this was not possible. With a more in-depth study, the rolling window regression could be performed for all five market-beta portfolios.

Finally, a limitation regarding the application of these results could also be apparent. Since these results were obtained using data on the US stock market, the results are also the most applicable to the US stock market. Even by applying the results to the US stock market with caution, the results could be of not much economic use. Also, the results obtained from this study could be detrimental if applied to other stock markets. For example, if applied to the rising Chinese stock markets, results could be detrimental due to completely different regulations, differing investors' attitudes and economic environment and so on. Therefore, the scope of application of these results is minimal at best.

#### **5.4 Future Studies**

Firstly, in future studies the causes of the time-varying nature of the underlying components driving portfolio returns could be investigated further. This was not addressed in detail in this thesis. Therefore, it would bring clearer insight into how and why the underlying components change over time. Eventually, with a clearer picture on how and why the underlying components change over time, a more accurate prediction of how the components will change in the future can be made. Finally, these more accurate predictions can be used more efficiently in hedging as well as investors attempting to profit from market timing.

Furthermore, future studies could investigate the time-varying nature of the underlying components of portfolios sorted on different characteristics (other than market-beta). For example, some other univariate sorts could include: size,

B/M, operating profit and investment. Besides univariate sorts, a more in-depth research paper could consider bivariate sorts such as: size and B/M, size and operating profitability, size and investment, B/M and operating profitability, B/M and investment as well as operating profit and investment. It would be interesting to see how the underlying components driving portfolio returns vary across the different sorts as well as over time.

Finally, although there are some studies done in other markets worldwide, future studies could produce a concrete comparison of the development of the underlying components of portfolio returns across markets. For example, a study comparing the development of the underlying components of portfolio returns of developed markets (US or Europe) vs emerging markets (South-East Asia or Latin America). The importance of certain underlying components and how they differ from markets could be of interest.

## **5.5 Ethical Issues**

An important ethical issue related to this study could result from blind over-reliance on historical results. For example, if a financial advisor were to recommend certain investments based off encouraging results from the rolling window regression performed earlier. If a period returns highly positive and significant coefficients for example SMB, a financial advisor might encourage unknowledgeable clients to invest heavily in small cap firms. However, as seen in the rolling window regression, the coefficients really do vary significantly over time. Therefore, blind over-reliance on historical results is against the interests of the client and thus could pose an ethical issue in the form of conflict of interests.

Since financial markets will never be perfectly predictable, caution and a certain amount of skepticism must be applied when utilizing models such as this. On top of informing the client about the uncertainty associated with the model, another highly regarded model could be used to cross-reference and verify the predictions imposed by the model used in this paper.

Another important ethical issue related to this study could be to insist the client to invest into a more active strategy. For example, a financial advisor could



recommend certain investment strategies with high turnover rates and high trading volumes such as a reversal strategy or another momentum-based strategy. Such strategies require frequent trading by the financial advisor and therefore generate higher fee income for them at the expense of the client. This kind of conflict of interest can appear if the advisor is being unethical and does not disclose all of the relevant information about their intentions to the client. Again, the most relevant solution could be to consult another financial advisor regarding the suggestions on active investing to uncover the true motives behind it.

## 6. Conclusion

Both the maximum time period regression of the five market-beta sorted portfolios and the rolling window regression of the long-short portfolio illuminated striking revelations in the underlying components driving portfolio returns. The maximum time period regression consisted of a cross-sectional analysis of the five market-beta sorted portfolios against the nine-factor regression model. While, the rolling window regression consisted of the long-short portfolio, which attempts to pinpoint the time-varying nature of the underlying components between the two extremes, the high market-beta portfolio's excess returns and the low market-beta portfolio's excess returns.

The maximum time period regression results in Table I displayed significant increasing and decreasing trends from the low-beta portfolio to the high-beta portfolio. Most trends were highly significant even after adjusted for possible errors in the data such as heteroscedasticity and autocorrelation. These statistically significant trends could be explained by logical characteristics and actions associated with firms in the specific category.

For example, the most important trends included the consistently increasing coefficient for the market premium factor (Mkt-RF) and the SMB factor as well as the consistently decreasing trend for the CMA factor. The statistically significant increasing trend in the market premium factor results from the original sort on the portfolios being done on market-betas, which basically measures the same exposure as the market premium factor used in the regression. The statistically significant increasing trend in the SMB factor is due to the higher volatility in returns in small cap firms, which tend to have higher market-betas. Finally, the consistently decreasing trend in the CMA factor is because of the tendency for small cap firms, who tend to have a high market-beta, to invest more aggressively due to the early stage of the company life cycle that they are in.

Not only did the maximum time period regression results in Table I display significant trends, however the lack of a significant trend with certain regressors is of just as much importance. For instance, the lack of a significant trend in the

constant term in the regression paired with insignificant robust t-statistics indicate that there is not a noticeable premium that is left unexplainable by the model used in the regression. This is further enhanced by the r-squared values and the adjusted r-squared values being of about 0.90 and over. This means that the model is able to explain about 90% of the variation in the portfolio returns, which makes it a very effective model for the data.

The rolling window regressions performed on the long-short portfolio mainly revealed the time-varying nature of the underlying components driving portfolio returns. The time-varying nature of the underlying components were mainly attributed to significant financial crises that occurred at the time. For example, the 1975-1982, 1990-2000 and 2006-2018 turbulent times were attributed to the 70s energy crisis, dot-com bubble and the US housing market bubble respectively. Since most of the coefficients were not statistically significant during this time, it is difficult to meaningfully comment on their evolution throughout the time period. However, most of the insignificance is attributable to the few amounts of data points used in the rolling window regression.

The coefficients that were statistically significant despite the small amount of data points, revealed that they were truly distinct from zero. The two most significant regressors in the rolling window regression are the Mkt-RF factor as well as the SMB factor.

Overall the results were in line with the previous literature by Fama and French 2015 regarding the importance of the Fama-French five factors, however opposed almost all of the remaining literature regarding additional variables. Therefore, the model most likely suffered from a statistical error of over-fitting and the exposure of the additional variables were already at least partially captured by the Fama-French five factors.

The rolling window regression also revealed how volatile and time-varying the coefficients and their robust t-statistics are. Therefore, the added predictability that this model brings to predicting expected stock portfolio returns is minimal. Therefore, in order to profit from market timing and hedge more efficiently, more consistent and accurate models would need to be obtained.

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